

ANALYSIS OF HEAT EXCHANGE AND AUTOMATIC CONTROL OF THE SYSTEM BASED ON THE FOURIER AND LUMPED MODELS IN THE PROCESS OF COOLING GRANULES IN A WATER BATH

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Abstract. This article examines the issues of automatic control of the process of cooling granulated products in a water bath. To describe the heat exchange process, the Fourier model and the simplified Lumped model with aggregate parameters are analyzed. The advantages and areas of application of both models are compared, and their effectiveness in creating an automatic control system is evaluated.

Key words: granule, cooling process, automatic control, Fourier model, Lumped model, heat exchange, mathematical modeling, energy consumption.

Introduction. In industrial processes, the production of granulated products and their high-quality cooling is one of the important stages. In particular, the process of cooling in a water bath has high efficiency, which directly affects the quality of the product [1]. Mathematical models expressing the laws of heat transfer are necessary for effective process control [2].

Problem situation. The following parameters are important in controlling the granule cooling process: 1. water temperature, 2. flow rate, 3. granule size, 4. inlet temperature [3]. In practice, the following problems are observed in the process of cooling granules: 1. uneven cooling of granules; 2. differences in internal and external temperature; 3. Increased energy consumption; 4. slow operation or insufficient accuracy of the control system [4]. Traditional management methods often do not fully reflect the real process, as a result of which product quality decreases.

Solution to the problem. Mathematical modeling of the heat exchange process plays an important role in solving these problems.

The Fourier model accurately describes the temperature distribution inside the granules:

$$\frac{\partial T}{\partial t} = a \nabla^2 T \quad (1)$$

This model provides high accuracy and takes into account the internal thermal gradients in the granule. However, due to the complexity of calculations, it is difficult to use in real-time management.

The Lumped model is represented by the following equation:

$$\frac{dT}{dt} = - \frac{hA}{\rho c V} (T - T_{water}) \quad (2)$$

This model characterizes the granule with a single temperature and is very simple in terms of calculation. Therefore, it is widely used in automatic control systems [5]. As an optimal solution: 1. when designing the system, a deep analysis is carried out based on the Fourier model; 2. The Lumped model is used in real-time management; 3. The temperature is maintained constant using a PID controller.

Initial temperature: $T_0 = 120^\circ\text{C}$;

Water temperature: $T_{water} = 25^\circ\text{C}$;

Granule radius: $r = 0,005 \text{ m}$;

Density: $\rho = 1200 \text{ kg/m}^3$;

Heat capacity: $c = 1500 \text{ J / (kg} \cdot ^\circ\text{C)}$;

Heat transfer coefficient: $h = 500 \text{ W / (m}^2 \cdot ^\circ\text{C)}$.

Calculation of geometric quantities

For the sphere:

$$V = \frac{4}{3} \pi r^3 = \frac{4}{3} \cdot 3,14 \cdot (0,005)^3 \approx 5,24 \cdot 10^{-7} \text{ m}^3 \quad (3)$$

$$A = 4 \pi r^2 = 4 \cdot 3,14 \cdot (0,005)^2 \approx 3,14 \cdot 10^{-4} \text{ m}^2 \quad (4)$$

Lumped model

Formula:

$$T(t) = T_{water} + (T_0 + T_{water})e^{-\frac{hA}{pcV}t} \quad (5)$$

Let's calculate the indicator:

$$\frac{hA}{pcV} = \frac{500 \cdot 3,14 \cdot 10^{-4}}{1200 \cdot 1500 \cdot 5,24 \cdot 10^{-7}} \approx 0,166 \text{ s}^{-1} \quad (6)$$

Therefore:

$$T(t) = 25 + (120 - 25)e^{-0,166t} \quad (7)$$

Time (s)	Temperature (°C)
0	120°C
5	$25 + 95 \cdot e^{-0,83} \approx 66^\circ\text{C}$
10	$\approx 44^\circ\text{C}$
20	$\approx 30^\circ\text{C}$
30	$\approx 27^\circ\text{C}$

The results are as follows: in 20-30 seconds, the granules reach almost the water temperature.

Biot count check

$$Bi = \frac{h \cdot r}{k} \quad (8)$$

Suppose: $k = 0,5 \text{ W}/(\text{m} \cdot ^\circ\text{C})$

$$Bi = \frac{500 \cdot 0,005}{0,5} = 5$$

Bi > 0,1, therefore: The Lumped model is inaccurate and the Fourier model should be used

Fourier model

According to Fourier's model, there is a temperature gradient inside the granule, so the center's temperature cools slower than the surface.

Point	Temperature
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Surface	$\approx 40^{\circ}\text{C}$
Center	$\approx 70^{\circ}\text{C}$

The Lumped model calculates quickly and simply, while the Fourier model is precise but complex. When the number of Biotes is large, Lumped gives an error. In automatic control, quick calculation is in Lumpet, and accurate analysis is in Fourier.

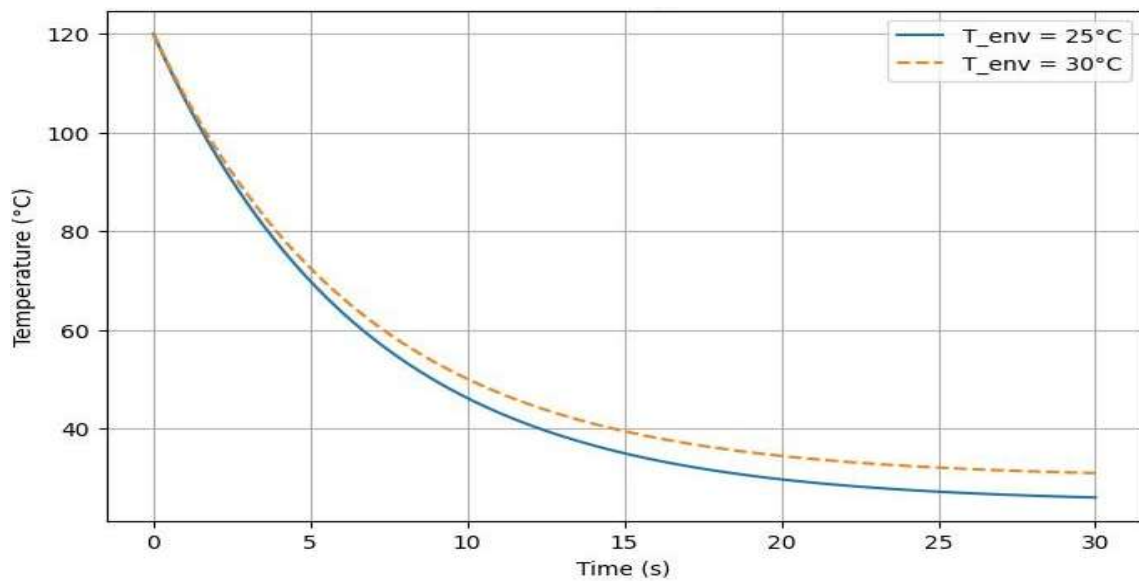


Figure 1. Granule cooling process

The blue line at 25°C water cooled quickly, and the dashed line at 30°C water cooled slowly.

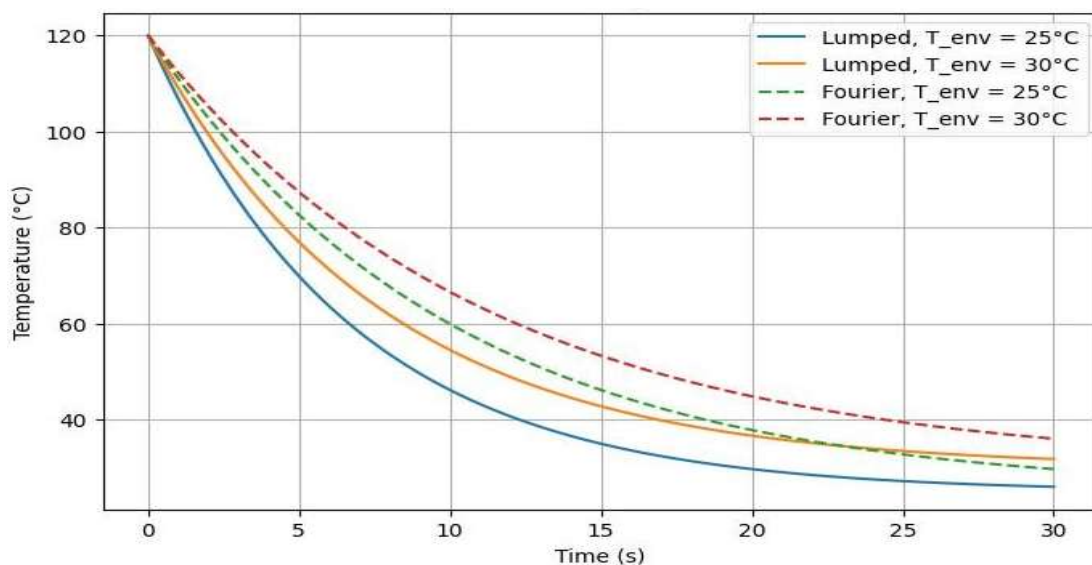


Figure 2. Granule cooling process $T_{\text{water}} = 25^{\circ}\text{C}$ and 30°C , Lumped and Fourier
Blue lines $T_{\text{water}} = 25^{\circ}\text{C}$, thick line Lumped, slit line Fourier. Red lines $T_{\text{water}} = 30^{\circ}\text{C}$, thick line Lumped, slit line Fourier. The Lumped model takes into account rapid cooling, while the Fourier model takes into account slow cooling, internal temperature.

Conclusion. Mathematical modeling is an important tool for effective control of the granule cooling process in a bath. The Fourier model provides high accuracy, while the Lumped model is distinguished by its simplicity. Their combined application increases the efficiency of the automatic control system and improves product quality. The control system is usually built on the basis of a PID controller. The Lumped model is used for operational control, while the Fourier model is used for in-depth system analysis.

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