DEVELOPING HIGH-LEVEL THINKING SKILLS IN FUTURE TEACHERS THROUGH PROBLEM-BASED LEARNING AND MATHEMATICAL TASKS

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Abstract

The preparation of future teachers requires more than the acquisition of ready-made mathematical procedures. Prospective teachers must learn to formulate problems, select and justify strategies, interpret evidence, detect misconceptions, and redesign explanations for learners. This conceptual-methodological paper examines how problem-based learning (PBL) and mathematical research tasks can be integrated to develop these higher-level thinking skills.

Keywords

problem-based learning; future teachers; higher-order thinking; mathematical inquiry; research tasks; pedagogical content knowledge; formative assessment

Introduction

Contemporary teacher education must prepare graduates to act in situations for which no single algorithm is available. A future teacher is expected to interpret unfamiliar classroom evidence, diagnose students' reasoning, compare alternative representations, justify instructional choices, and improve a lesson after observing its effects [1]. These actions involve analysis, evaluation, creation, and metacognitive regulation rather than the reproduction of facts. In mathematics, the demand is especially visible: knowing a formula is not equivalent to understanding why it works, when it is useful, how learners may misinterpret it, or how an alternative solution can be validated.

International frameworks reinforce this shift. The OECD defines mathematical literacy through the capacity to reason mathematically and to formulate, employ, and interpret mathematics in varied real-world contexts [2]. Its creative-thinking framework also treats creativity as a developable competence expressed through generating diverse ideas, generating original ideas, and evaluating and improving ideas. Consequently, teacher education should provide repeated opportunities to make conjectures, test evidence, communicate arguments, and revise incomplete solutions rather than reserve inquiry for exceptional students.

The work of Yunusova provides a locally relevant methodological foundation for this orientation. Her studies connect the mastery of mathematical concepts with active participation, creative and innovative thinking, structured cognitive activity, interactive practice, and continuous feedback [3, 4, 5, 6, 7, 8, 9]. Her research on educational projects and pedagogical qualimetry additionally emphasizes that complex professional competence requires both authentic activity and systematic assessment [4, 6]. Building on these ideas and international evidence, this paper aims to develop a practical framework for cultivating high-level thinking in future teachers through PBL and mathematical research tasks.

Higher-level thinking is used here as an integrated construct comprising four mutually dependent capacities. Analytical thinking decomposes a problem, identifies conditions, and connects representations. Critical thinking evaluates assumptions, the reliability of evidence, and the validity of an argument. Creative thinking generates multiple approaches, examples, models, or pedagogical explanations. Metacognitive thinking plans, monitors, and evaluates one's own reasoning. In the revised taxonomy of educational objectives, these capacities correspond broadly to analysing, evaluating, and creating, but in professional teacher learning they must operate together [1].

This integration matters because future teachers work with two levels of reasoning. At the first level, they solve a mathematical problem and justify the result. At the second, they examine the solution pedagogically: Which prerequisite ideas are involved? What errors are likely? Which questions would reveal understanding? How might the task be adapted for different learners? The second level transforms subject knowledge into pedagogical content knowledge. It also prevents inquiry from becoming an attractive activity that is disconnected from the future teacher's professional decisions.

Yunusova's three-stage view of mathematics lessons is compatible with such development because it assigns different methods to successive phases of cognitive activity [5]. Her active-approach study highlights interaction, student activity, individual and group experience, and feedback as central conditions of practical mathematics instruction [8]. These conditions are not supplementary techniques. They create the social and cognitive environment in which learners must externalize, test, and reconstruct their thinking.

PBL begins with a complex problem that organizes the need for knowledge. Learners clarify the situation, identify what they know and need to learn, gather and evaluate information, develop possible solutions, and reflect on both the outcome and the process. The teacher structures participation and provides strategic scaffolding without converting the problem into a sequence of instructions. A meta-analysis of 50 higher-education studies involving 5,210 participants found an overall positive relationship between PBL interventions and critical-thinking outcomes, while also showing that effects depend on design and implementation variables [10]. Thus, PBL should not be treated as a label that automatically guarantees higher-order thinking.

Evidence from mathematics teacher education is similarly promising but nuanced. Bosica, Pyper, and MacGregor [11] reported meaningful changes in preservice secondary mathematics teachers' beliefs and orientations when PBL was incorporated

into a methods course. Martin [12] found that a closed-loop PBL approach was as effective as teacher-directed instruction on a multiple-choice measure of mathematics pedagogical content knowledge; importantly, PBL participants also enacted their knowledge through open-ended planning and simulated teaching. This cautions against judging an inquiry pedagogy only with instruments designed mainly for declarative knowledge.

Mathematical research tasks complement PBL by making inquiry processes visible. Unlike routine exercises, a research task asks learners to formulate sub-questions, search for patterns, generate examples and counterexamples, vary conditions, represent relationships, make conjectures, and construct an argument. Problem posing is particularly valuable because it requires the learner to reorganize mathematical experience into a meaningful new problem. A recent meta-analysis confirms that problem-posing competence can be deliberately developed, although effective interventions vary and require carefully designed support [13].

The proposed cycle connects PBL, mathematical inquiry, and professional reflection. It is designed for a two- to four-week module in courses such as mathematics teaching methodology, elementary mathematics, geometry, statistics, or assessment. The phases are recursive: new evidence may require learners to return to an earlier decision.

Phase 1 - Problem encounter and orientation. The instructor presents an authentic professional situation with incomplete information and more than one defensible response. For example, future teachers may be asked to redesign a textbook task that produces correct answers but little explanation. Individually, students record initial interpretations and assumptions; in groups, they identify the mathematical and pedagogical tensions. This phase activates prior knowledge and creates a reason to learn.

Phase 2 - Research-question formulation. Groups convert the broad situation into investigable questions. They distinguish questions about mathematical structure from

questions about teaching and learning. A useful protocol asks: What varies? What remains invariant? What evidence would support a claim? What learner response would disconfirm our interpretation? The instructor checks that questions are neither trivial nor too broad.

Phase 3 - Mathematical investigation. Students produce examples, counterexamples, diagrams, tables, symbolic representations, simulations, or small data sets. They compare strategies and document failed attempts. Yunusova's work on frames for mastering mathematical concepts is relevant here: structured organizers can help learners connect definitions, properties, examples, non-examples, and applications without replacing the inquiry itself [3,4].

Phase 4 - Argumentation and revision. Groups present claims and evidence in a research seminar. Peers challenge hidden assumptions, request justification, and propose alternative interpretations. The presenting group revises its solution and makes the chain of reasoning explicit. Revision is essential because high-level thinking is demonstrated not only by producing an idea but also by improving it in response to evidence.

Phase 5 - Pedagogical translation and metacognitive reflection. Students convert their mathematical findings into a lesson fragment, diagnostic question, assessment rubric, or set of prompts for school learners. They then write a brief reflection identifying a turning point in their reasoning, a rejected assumption, and a principle they can transfer to another topic. This closed-loop feature links inquiry to future classroom practice.

A high-quality task must be mathematically accessible at the entry point but expandable in depth. It should contain productive uncertainty: the solution path is not stated, yet the problem remains sufficiently bounded for learners to make progress. Tasks are stronger when they permit several representations, invite generalization, and require justification. For future teachers, an additional layer should ask how the mathematics can be taught, assessed, or adapted.

One example is the task “Same area, different perimeter.” Groups first construct all rectangles with integer side lengths and area 36 square units. They organize the cases, determine the minimum and maximum perimeter under the stated conditions, and explain why their list is complete. They then remove the integer restriction and investigate how the perimeter changes for positive real side lengths. Finally, they design questions that could guide school learners from numerical cases toward the general result. The task combines pattern search, optimization, representation, proof, and pedagogical design. It also generates useful extensions: fixed perimeter and variable area, rectangles versus other quadrilaterals, or the effect of measurement error.

Three safeguards are necessary. First, openness should not mean vagueness: the intended concepts and acceptable evidence must be specified by the instructor. Second, collaboration should preserve individual accountability through research notes, exit explanations, or oral questioning. Third, digital tools should serve reasoning. A spreadsheet or dynamic-geometry system is useful when it helps generate cases or test conjectures, but students must still explain why observed patterns should hold.

Assessment should be aligned with the processes the approach seeks to develop. If only the final numerical answer is graded, students will rationally avoid documenting uncertainty, comparison, and revision. Author [6] describes assessment as a feedback component of educational quality management, while her work on modular-rating systems emphasizes transparent learning programmes, staged control, and independent study [7]. These principles support a combination of diagnostic, formative, and summative evidence.

At the beginning of a module, a short diagnostic task can reveal whether students generate examples, justify claims, and monitor assumptions. During inquiry, the instructor reviews research questions, logs, interim representations, and peer feedback. At the end, the product is assessed together with an individual explanation and reflection.

Table 1 offers a compact analytic framework. Scores may be assigned from 0 (not demonstrated) to 3 (independent and well-justified), but descriptors and exemplars should be discussed before the task begins.

Dimension	**Evidence expected in a strong performance**
Mathematical reasoning	Correct concepts; connected representations; explicit assumptions; valid justification.
Inquiry design	Focused research questions; systematic cases; purposeful use of examples and counterexamples.
Use of evidence	Claims are matched to evidence; limitations are recognized; revisions respond to critique.
Pedagogical translation	Anticipates learner strategies and misconceptions; designs prompts and assessment evidence.
Metacognition and collaboration	Documents decisions, role contributions, failed approaches, and transferable learning.

Table 1. Analytic criteria for assessing mathematical inquiry and pedagogical thinking

The proposed model has three implications for teacher education. First, higher-level thinking should be embedded in disciplinary work rather than taught as a generic list of skills. Critical evaluation in mathematics depends on definitions, representations, and standards of proof; creative thinking depends on sufficient knowledge to produce meaningful alternatives. Second, the unit of design should be a sequence of connected tasks. A single open problem may stimulate interest, but stable habits of inquiry require repeated cycles of formulation, investigation, argumentation, and reflection.

Third, teacher educators need to balance autonomy with scaffolding. Excessive direction reduces a research task to an exercise, whereas insufficient guidance may reward only students who already possess strong inquiry strategies. Productive scaffolds include worked comparisons rather than worked answers, question stems for challenging claims, role rotation in groups, interim conferences, and partially completed representations. Scaffolds should be faded as students gain control of the process.

The synthesis also identifies a methodological caution. Positive outcomes attributed to PBL may vary with the problem type, duration, tutor expertise, group size, and assessment instrument [10]. Future empirical work should therefore report the intervention precisely and use multiple measures: performance tasks, argument quality, reflective writing, and observation of teaching decisions. Longitudinal studies are especially needed to determine whether inquiry habits developed at university transfer into school practice.

Problem-based learning and mathematical research tasks can provide a coherent environment for developing the analytical, critical, creative, and metacognitive capacities required of future teachers. Their value lies not in replacing systematic instruction, but in reorganizing knowledge around professionally meaningful problems and making reasoning open to evidence, dialogue, and revision. The five-phase cycle proposed in this paper links mathematical investigation with pedagogical interpretation: future teachers encounter a problem, formulate research questions, investigate, defend and revise claims, and translate the result into teachable action.

This approach extends Yunusova's emphasis on active mathematics learning, structured cognitive activity, conceptual construction, educational projects, and feedback-oriented assessment. For implementation, teacher educators should begin with a small number of carefully bounded research tasks, make success criteria explicit, and evaluate both the mathematical product and the learning process. When these conditions are met, future

teachers do not merely learn solutions; they learn how to inquire, justify, notice, and design—capacities that can shape the quality of their later classroom practice.

References:

1. Anderson, L. W., & Krathwohl, D. R. (Eds.). (2001). A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives. Longman.

2. OECD. (2023). PISA 2022 assessment and analytical framework. OECD Publishing. https://doi.org/10.1787/dfe0bf9c-en

3. Yunusova, G. A. (2022a). Methods of effective use of some frames, methods in mastering mathematical concepts in practical lessons from mathematics. Asian Journal of Research in Social Sciences and Humanities, 12(4), 412–415. https://doi.org/10.5958/2249-7315.2022.00207.6

4. Yunusova, G. A. (2022b). Educational projects as a means of a comprehensive assessment of the methodological training of future teachers. Science and Innovation, 1(8), 2448–2452.

5. Yunusova, G. A. (2023a). Methods that develop the didactic structure of students' cognitive activity in three stages of a math lesson. International Journal of Pedagogics, 3(5), 108–112. https://doi.org/10.37547/ijp/volume03issue05-22

6. Yunusova, G. A. (2023b). The test as a qualimetric tool to assess the quality of students' knowledge. Educational Research in Universal Sciences, 2(1 Special), 112–116. https://erus.uz/index.php/er/article/view/1767

7. Yunusova, G. A. (2023c). Methods of the modular-rating system of teaching and control as a strategy for improving the monitoring of the learning process of students. Educational Research in Universal Sciences, 2(2), 264–268. https://erus.uz/index.php/er/article/view/1724

8. Yunusova, G. A. (2024). Improving the methodology of passing practical classes in math on the basis of an active approach. Open Herald: Periodical of Methodical Research, 2(3), 65–67. https://academiaone.org/index.php/6/article/download/651/539/1190

9. Yunusova, G. A. (2025). Methodology of fully mastering the basics of math analysis. Web of Teachers: Inderscience Research, 3(3), 212–215. https://webofjournals.com/index.php/1/article/view/3640

10. Liu, Y., & Pásztor, A. (2022). Effects of problem-based learning instructional intervention on critical thinking in higher education: A meta-analysis. Thinking Skills and Creativity, 45, 101069. https://doi.org/10.1016/j.tsc.2022.101069

11. Bosica, J., Pyper, J. S., & MacGregor, S. (2021). Incorporating problem-based learning in a secondary school mathematics preservice teacher education course. Teaching and Teacher Education, 102, 103335. https://doi.org/10.1016/j.tate.2021.103335

12. Martin, D. A. (2022). The impact of problem-based learning on pre-service teachers' mathematics pedagogical content knowledge. Australian Journal of Teacher Education, 47(4), 55–74. https://doi.org/10.14221/ajte.2022v47n4.4

13. Zhang, L., Stylianides, G. J., & Stylianides, A. J. (2024). Enhancing mathematical problem posing competence: A meta-analysis of intervention studies. International Journal of STEM Education, 11, 48. https://doi.org/10.1186/s40594-024-00507-1