SOLUTIONS OF NEWTON INTERPOLATION PROBLEMS USING THE PYTHON PROGRAMMING LANGUAGE

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Abstract: This article investigates the theoretical and practical aspects of constructing the Newton interpolation polynomial, a key method for function approximation. Furthermore, it demonstrates the steps for determining coefficients by constructing a system of linear equations based on given datasets. In the practical section, Python programming code and algorithms for calculating the Newton polynomial using the Gauss elimination method are provided

Keywords: Newton interpolation, interpolation nodes, polynomial, system of linear equations, Gauss elimination, Python, algorithm design.

NYUTON INTERPOLYATSIYA MASALALARINI PYTHON DASTURLASH TILIDA YECHISH

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Annotatsiya: Ushbu maqolada funksiyalarni yaqinlashtirishning muhim usullaridan biri bo‘lgan Nyuton interpolatsiya ko‘phadini qurish masalasi nazariy va amaliy jihatdan tadqiq etilgan. Maqolaning amaliy qismida Gauss usuli yordamida Nyuton ko‘phadini hisoblovchi Python dasturiy kodi va algoritmlari taqdim etilgan.

Kalit so‘zlar: Nyuton interpolatsiyasi, interpolatsiya tugunlari, ko‘phad, chiziqli tenglamalar sistemasi, Gauss usuli, Python, algoritmlash.

Amaliy hisoblashlarda Nyutonning interpolyatsiyalashga oid ikki xil formulasi keng qo‘llaniladi. Nyutonning birinchi interpolyatsiya formulasi yordamida $y = f(x)$

funksiyaning qiymatini x_0 boshlang‘ich nuqtaga yaqin joylarda aniqlash qulay hisoblanadi. Shu sababli bu formula jadvalning bosh qismidagi qiymatlar bilan ishlashda samarali bo‘ladi, biroq jadval oxiridagi nuqtalar uchun hisoblash biroz noqulaylik tug‘diradi. Nyutonning ikkinchi interpolyatsiya formulasi esa, aksincha, kesmaning oxirgi nuqtasi x_n dan boshlab orqaga qarab hisoblashga asoslanadi.

Aytaylik , f(x) funksiya $x_0, x_1, \dots, x_n$ tugun nuqtalardagi

$$y_0 = f(x_0), \quad y_{1=}f(x_1), \dots, y_n = f(x_n)$$

Qiymatlar bilan berilhan bo’lsin,

$$N_n(x_0) = y_0, N_n(x_1) = y_1, \dots, N_n(x_n) = y_n$$

Tengliklarni qanoatlantiruvchi $N_n(x)$ ko’phadni qurish talab qilinmoqda. n-darajal i ko’phadning mavjudlik va yagonalik shartidan kelib chiqib, uni quyidagi tenglama ko’rinishida ifodalanadi.

$$N_n(x_i) = d_0 + d_1(x_i - x_0) + d_2(x_i - x_0)(x_i - x_1) + \cdots$$
$$+ d_n(x_i - x_0)(x_i - x_1) \dots .(x_i - x_{n-1}) = f(x_i), i = 0..n$$

Nyuton interpolyatsiya ko’phadi

$N_n(x)$ ning noma’lum d_i, i=0,1,...n koeffisentlarini interpolyatsiya shartlaridan topamiz:

$$N_n(x_i) = d_0 + d_1(x_i - x_0) + d_2(x_i - x_0)(x_i - x_1) + \cdots$$
$$+ d_n(x_i - x_0)(x_i - x_1) \dots .(x_i - x_{n-1}) = f(x_i), i = 0..n$$

Biz quyi chap uchburchak matritsali tenglamalar sistemasini olamiz:

$$d_0 = f(x_0),$$
$$d_0 + d_1(x_{1-}x_0) = f(x_1),$$
$$d_0 + d_1(x_2 - x_0) + d_2(x_2 - x_0)(x_2 - x_1) = f(x_2),$$

....

$$d_0 + d_1(x_n - x_0)(x_n - x_1) + \cdots + d_n(x_n - x_0) \dots (x_n - x_{n-1}) = f(x_n).$$

Bu quyi chap uchburchak matritsalichiziqli tenglamalar sistemasi eng birinchi tenglamadan boshlab yechiladi:

$$d_0 = f(x_0), \qquad d_1 = \frac{[f(x_1) - f(x_0)]}{x_1 - x_0} = f[x_0, x_1],$$

$f(x_0) + f[x_0, x_1](x_2 - x_0) + d_2(x_2 - x_0)(x_2 - x_1) = f(x_2) \Rightarrow f[x_0, x_1, x_2]$.

Va matematik induksiya bo'yicha, $d_1 = f[x_0, x_1, \dots, x_k]$, k$\leq n$.

Demak, interpolyatsiya ko'phadining Nyuton ko'rinish quyidagicha bo'ladi.

$$N_n(f\colon x) = f(x_0) + f[x_0, x_1](x - x_0) + \cdots + f[x_0, \dots, x_n](x - x_0) \dots (x - x_{n-1})$$

Yuqoridagi tenglama Nyutonning 1-interpolyatsiya formulasi deyiladi.

Agar $x_1 - x_0 = \cdots = x_n - x_{n-1}$ $bo'lsa \frac{x - x_0}{h} = t$ deb ushbu formulani hosil qilamiz:

$$N_n(f\colon x) = f(x_0) + hf[x_0, x_1]t + \cdots + h^n f[x_0, \dots, x_n]t(t-1) \dots (t - n + 1), x = x_0 + th$$

Nyutonning 1-interpolyatsiya formulasida interpolyatsiya x_0 nuqtadan boshlanadi. Nyutonning 2-interpolyatsiya formulasida esa interpolyatsiya x_n nuqtadan boshlanadi:

N_n(f;x)=$f(x_n) + f[x_{n-1}, x_n](x - x_n) + \cdots + f[x_0, \dots, x_n](x - x_n) \dots (x - x_1)$

Tenlamani keltirib chiqarishda paydo bo`ladigan chiziqli tengamalar sistemasi, yuqori o'ng burchak sistemadan iborat bo'ladi va noma'lumlar $d_n, d_{n-1}, \dots, d_1, d_0$tartibda topib boriladi. Demak Nyutonga chiziqli tenglamalar sistemasi va ularning yechilishi mumkin bo'lgan sodda hollari ma'lum bo'lgan.

Masalaning qo'yilishi:

X	2	1	0	-1
Y	*4*	*5*	*6*	*2*

Nyuton interpolyatsion ko'phadini tuzing.

Masalaning Python dasturlash muhitidagi yechimi

Dastur kodi;

```
Import numpy as np
defgaus_elimination(A,b):
n=len(b)
A=A.astype(float)
b=b.astype(float)
for I in range(n):
    #dioganal elementni 1 ga aylantiramiz
pivot=A[i,i]
   A[i]=A[i]/pivot
b[i]=b[i]/pivot
for j in range(i+1, n):
factor=A[j,i]
A[j]=A[j]-factor*A[i]
b[j]=b[j]-factor*b[i]
#orqaga qaytish usuli (Back Substitution)
x=np.zeros(n)
for I in range(n-1,-1,-1):
x[i]=b[i]-np.dot(A[i, i+1], x[i+1:])
return x
x_vals=np.array([2, 1, 0, -1])
y_vals=np.array([4, 5, 6, 2])
A=np.array([ [1, 2, 4, 8] [1, 1, 1, 1] [1, 0, 0, 0] [1, -1, 1, -1]])
b=y_vals
coeffs=gauss_elimination(A, b)
print("Nyuton interpolyatsiya ko'phadining koeffisentlari:")
```

```
print(f'a0={coeffs[0]}")
print(f'a1={coeffs[1]}")
print(f'a2={coeffs[2]}")
print(f'a3={coeffs[3]}")
print("\nInterpolyatsiya qilingan ko'phad:")
print(f'P(x)= {coeffs[0]} +{coeffs[1]}*x+{coeffs[2]}*x^2+{coeffs[3]}*x^3)
```

Natija:

Nyuton interpolyatsiya ko'phadining koeffisentlari:

a0=6.000000000000001

a1=0.6666666666666661

a2=-2.5

a3=0.8333333333333334

Interpolyatsiya qilingan ko'phad:

P(x)=6.000000000000001+0.6666666666666661*x+-2.5*x^2+0.8333333333333334*x^3[1].

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