The ICM (Inductive Contextual Modelling) Algorithm in Mathematics Teaching for Chemistry Students

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Abstract

Chemistry students frequently struggle to use abstract mathematics to reason about chemical phenomena, even when their procedural mathematical skills are adequate. This paper proposes and conceptually examines an Inductive Contextual Modelling (ICM) algorithm for teaching mathematics to chemistry students. The algorithm integrates three well-established research traditions: inductive teaching as systematised by Prince and Felder, the emergent-modelling design heuristic of Realistic Mathematics Education developed by Gravemeijer, and the mathematical modelling cycle of Blum and Leiß. ICM is defined as a seven-step instructional sequence that begins with a chemical phenomenon, guides students through informal ("model-of") representations toward formal ("model-for") mathematical structures, and returns to chemical interpretation through validation. The paper articulates the theoretical foundations of ICM, formalises its seven steps, illustrates them on a chemical-kinetics example (the steady-state approximation), and discusses the algorithm's coherence with existing empirical evidence on calculus-in-chemistry instruction and mathematical-modelling competence. Testable hypotheses for subsequent quasi-experimental research are outlined.

Keywords: inductive teaching; contextual learning; mathematical modelling cycle; chemistry education; emergent modelling; mathematics for chemists.

1. Introduction. Mathematics is both a tool and a language of chemistry. Stoichiometric proportions, equilibrium expressions, rate laws, thermodynamic potentials and quantum-mechanical operators all require students to translate fluently between

chemical phenomena and mathematical representations. Empirical research nevertheless consistently shows that this translation is a major source of difficulty. A recent investigation of chemistry students solving steady-state-approximation tasks in chemical kinetics, for example, documents that students struggle most not with executing mathematical operations but with the transitions between the chemical situation and its mathematical description — the very transitions that the mathematical modelling cycle is intended to articulate [1]. Reviews of calculus-in-chemistry research likewise emphasise that chemistry students often fail to connect derivatives with rate of change in kinetics, or integrals with accumulated quantities in thermodynamics, because the underlying mathematical concepts have been learned in decontextualised form [2].

Two influential traditions in science and mathematics education provide complementary responses to this problem. The first is inductive teaching. As systematised by Prince and Felder in their comprehensive review, inductive instruction inverts the traditional deductive sequence: rather than presenting a general principle followed by examples, the teacher begins with specific observations, cases or problems and helps students reconstruct or discover the underlying principle on the basis of evidence and need [3]. Inductive methods — including inquiry-based learning, problem-based learning and discovery learning — are consistently found to be at least as effective as, and often more effective than, traditional deductive methods for a broad range of learning outcomes [3]. The second tradition is the modelling perspective on mathematics education. Within Realistic Mathematics Education, Gravemeijer's emergent-modelling heuristic specifies how informal student-generated models of a contextual situation ("models of") can gradually evolve into formal mathematical objects ("models for") usable in further mathematical reasoning [4]. In parallel, Blum and Leiß have articulated a seven-step mathematical modelling cycle that explicitly distinguishes a situation

model, a real model, a mathematical model, mathematical results and a real-world interpretation, together with the cognitive transitions linking them [5].

A general meta-synthesis of context-based and STS approaches in science education demonstrates that situating concepts in authentic contexts improves both motivation and conceptual understanding without compromising attainment on conventional measures [6]. When such an extended modelling cycle is applied empirically to chemical-kinetics problem solving, additional sub-processes — deliberation, evaluation, activation of extra-mathematical resources — emerge as central to expert performance, and existing modelling cycles need to be extended accordingly [1].

Despite the convergence of these traditions, their explicit integration into a single, operationalisable instructional sequence for teaching mathematics to chemistry students has received limited attention. The present paper proposes such a sequence: the Inductive Contextual Modelling (ICM) algorithm. The algorithm is "inductive" in the sense of Prince and Felder [3], "contextual" in the sense of context-based learning [6] and chemistry-first curriculum design [7], and "modelling" in the sense of emergent modelling [4] and the Blum–Leiß modelling cycle [5]. ICM is intended as a teachable algorithm — a finite, ordered sequence of pedagogical actions — that lecturers and tutors can apply to a wide variety of mathematical topics arising in undergraduate chemistry, from rate laws and equilibria to spectroscopic data analysis.

The objectives of this paper are threefold: (1) to articulate the theoretical foundations of the ICM algorithm by drawing together inductive, contextual and modelling research; (2) to formalise the algorithm as a seven-step instructional cycle and to relate each step to known design principles and empirical findings; and (3) to illustrate the algorithm on a worked chemical-kinetics example and to derive empirically testable propositions for subsequent quasi-experimental research. The paper follows the IMRaD convention.

2. Methods

2.1 Research design

The study is conceptual–analytical. It develops an instructional algorithm on the basis of a structured narrative review of three converging bodies of research: inductive teaching in engineering and science education, emergent and mathematical modelling within mathematics education, and context-based learning and mathematics-in-chemistry instruction. The objective of the review was not to provide an exhaustive bibliometric mapping but to extract the design principles that the three traditions share and to formalise them as an ordered sequence of pedagogical actions.

2.2 Source selection

Sources were identified through searches in Google Scholar, ERIC, Scopus and ScienceDirect using combinations of the terms “inductive teaching”, “inquiry-based learning”, “emergent modelling”, “mathematical modelling cycle”, “context-based chemistry”, “mathematics for chemistry” and “chemical kinetics learning”. Priority was given to peer-reviewed journal articles, monographs and edited-volume chapters published between 1999 and 2024 by established academic publishers. Older foundational works were retained when their conceptual contribution remained current. Sources were screened in two stages: title-and-abstract screening for thematic relevance, followed by full-text reading to extract design principles, modelling-cycle components and reported empirical effects.

2.3 Analytical procedure

The selected sources were coded along three analytical dimensions: (i) the inductive dimension (degree to which instruction begins with specific cases rather than abstract principles); (ii) the contextual dimension (degree to which mathematical content is anchored in authentic chemical situations); and (iii) the modelling dimension (degree to which the instructional sequence supports the transition from informal situational

reasoning to formal mathematical structures and back). Recurring instructional steps that appeared across the three dimensions were synthesised into an ordered seven-step algorithm. The algorithm was then exemplified by mapping it onto a documented chemical-kinetics problem-solving scenario [1] in order to demonstrate operational feasibility.

2.4 Limitations of the method

Because the present study is conceptual rather than empirical, its conclusions take the form of testable propositions rather than verified findings. A formal systematic-review protocol was not adopted, which means that selection bias cannot be fully excluded. The illustrative example is drawn from one topical area (chemical kinetics); generalisation to other chemical subdisciplines requires further empirical work. These limitations are discussed further in section 4.

3. Results

3.1 Theoretical foundations of the ICM algorithm

Three theoretical commitments anchor the ICM algorithm. The first is the inductive commitment: instruction begins with a concrete chemical observation, case or problem, and the relevant mathematical structure is reconstructed by the students with guided support rather than presented in advance. This commitment follows directly from the comparative analyses of inductive teaching methods reported in [3], which show that inquiry-based, problem-based and discovery approaches produce equal or superior learning outcomes when compared with deductive instruction. The second is the contextual commitment: the chemical context is not a cosmetic wrapper around generic mathematics but the conceptual starting point and the criterion of interpretive adequacy. This commitment draws on the synthesis of context-based and STS approaches in [6] and on the chemistry-first curriculum perspective developed in [7]. The third is the modelling commitment: the transition from situation to formal mathematics is treated as

a sequence of discrete cognitive operations, each of which can be diagnosed and supported. This commitment follows the emergent-modelling heuristic of [4], in which student-generated representations gradually evolve from "models of" a particular situation into "models for" more general mathematical reasoning, and the seven-step modelling cycle of [5], which provides a fine-grained cognitive map of these transitions.

The integration of these three commitments responds to a specific empirical finding from the chemistry-education literature. In their study of university students solving steady-state-approximation tasks, Rodriguez et al. [1] show that an extended mathematical modelling cycle with explicit deliberation and evaluation steps is needed to capture how chemistry students actually move between chemical phenomena and mathematical representations. ICM operationalises this extended cycle into a teachable sequence.

3.2 The seven steps of the ICM algorithm

The ICM algorithm consists of seven ordered steps. They are summarised in Table 1 and elaborated below.

Table 1. The seven steps of the Inductive Contextual Modelling (ICM) algorithm.

Step	Stage	Pedagogical action	Theoretical anchor
1	Chemical phenomenon	The teacher presents a concrete chemical observation, experimental data set or qualitative phenomenon and elicits students' initial accounts of it.	Inductive trigger [3]

Step	Stage	Pedagogical action	Theoretical anchor
2	Situation model	Students construct an informal, qualitative representation of the phenomenon (verbal description, sketch, table, rough graph).	Situation model [5]; model-of [4]
3	Real model	The class collaboratively isolates the chemically relevant variables, assumptions and idealisations ("what is held constant; what varies; what is neglected").	Real model [5]
4	Mathematisation	Students translate the real model into mathematical objects: equations, derivatives, integrals, matrices, statistical estimators.	Horizontal mathematisation [4]; mathematisation [5]
5	Mathematical work	Students perform the required mathematical operations, with scaffolding	Mathematical work [5]

Step	Stage	Pedagogical action	Theoretical anchor
		calibrated to diagnosed prior knowledge.	
6	Chemical interpretation	Mathematical results are re-interpreted in chemical terms; their physical plausibility, units and limiting behaviour are checked.	Interpretation [5]; deliberation [1]
7	Validation and generalisation	Students validate the model against new data or limiting cases and reflect on its scope; the "model-of" becomes a "model-for" further chemical reasoning.	Validation [5]; model-for [4]

Step 1 — Chemical phenomenon. The cycle starts with a concrete chemical observation rather than with a definition or a formula. This is the inductive trigger in the sense of [3]: students encounter a need to know before they are told what they are supposed to know. Suitable phenomena include experimental kinetic traces, titration curves, spectral data and thermodynamic data tables.

Step 2 — Situation model. Students produce an informal representation of the phenomenon. In modelling-cycle terms this is the situation model, the learner's mental picture of the chemical situation [5]; in RME terms it is a model-of the specific situation,

still tightly tied to its referent [4]. The pedagogical goal at this step is not mathematical correctness but the surfacing of students' chemical intuitions, including their misconceptions.

Step 3 — Real model. The class moves from a global description to a structured account in which the chemically relevant variables, assumptions and idealisations are made explicit. This is the real model in the Blum–Leiß cycle [5]: a simplified, chemically meaningful description that is not yet mathematical.

Step 4 — Mathematisation. Variables and relationships identified in step 3 are translated into mathematical objects. In emergent-modelling terms, this is horizontal mathematisation, the move from a contextual representation toward a more general symbolic one [4]; in the Blum–Leiß cycle, it is the transition from the real model to the mathematical model [5]. The selection of mathematical content is therefore driven by the chemical situation rather than by a pre-existing mathematics syllabus.

Step 5 — Mathematical work. The mathematical model is manipulated: equations are solved, derivatives evaluated, integrals computed, regressions performed. This is the step in which students' mathematical fluency is most directly engaged, and where scaffolding calibrated to diagnosed prior knowledge is critical [3]. Computer-algebra systems, statistical software and spreadsheets are used where they correspond to genuine professional practice in chemistry.

Step 6 — Chemical interpretation. Mathematical results are translated back into chemical statements. Units, signs, orders of magnitude and limiting behaviour are checked for chemical plausibility. The empirical study of chemical-kinetics problem solving in [1] highlights that this interpretive step is precisely where chemistry students activate extra-mathematical resources and where deliberation — "does this make chemical sense?" — is most important.

Step 7 — Validation and generalisation. The model is confronted with new data, limiting cases, or alternative chemical systems. The pedagogical aim is the transition described by Gravemeijer [4] from a model-of a particular chemical situation to a model-for further chemical reasoning: students recognise the same mathematical structure across different chemical contexts and gain the ability to deploy it in novel problems.

3.3 An illustrative application: the steady-state approximation

The seven steps of ICM can be illustrated on the steady-state approximation in chemical kinetics, the same topical area investigated empirically in [1]. In step 1, students are presented with experimental concentration–time data for a reaction proceeding through a short-lived intermediate. In step 2, they describe qualitatively what they see: a reactant decaying, a product accumulating, an intermediate whose concentration rises and falls. In step 3, they articulate the chemical assumptions: the intermediate is highly reactive, its rate of formation rapidly approaches its rate of consumption, and its concentration changes are small relative to those of reactants and products. In step 4, the assumption "rate of formation ≈ rate of consumption" is mathematised by setting the time derivative of the intermediate's concentration approximately equal to zero, yielding an algebraic relation between the intermediate concentration and the reactant concentration. In step 5, the resulting expressions are manipulated to obtain a rate law in terms of measurable quantities. In step 6, the rate law is re-interpreted chemically: limiting cases (small or large reactant concentration) are checked against chemical intuition. In step 7, the same approximation is applied to a different reaction mechanism (for example, an enzyme-catalysed reaction in the Michaelis–Menten regime), demonstrating that the mathematical structure has become a general-purpose tool, i.e. a model-for in the sense of [4].

3.4 Anticipated effects of the algorithm

On the basis of the reviewed evidence, ICM is expected to produce differentiated gains across the cognitive, operational, motivational and reflective aspects of students' mathematical preparation. The inductive structure of the algorithm is associated with gains in conceptual understanding, transfer and student motivation in comparative studies [3]. The contextual anchoring is associated with improved motivation and conceptual learning [6] and with stronger alignment between mathematical content and the cognitive demands of chemistry [2, 7]. The modelling structure is associated with the development of mathematical modelling competence in the sense of [5] and, in the specific case of chemical kinetics, with more productive trajectories through the transitions between chemistry and mathematics [1]. The reflective step 7 hypothetically supports the transition from model-of to model-for [4], although this transition has been less studied empirically in the chemistry context and constitutes a central testable hypothesis for subsequent research.

4. Discussion

The ICM algorithm proposed in this paper brings together strands of research that are frequently developed in parallel rather than in dialogue. The inductive-teaching tradition [3] has produced strong evidence for the effectiveness of beginning with concrete cases, but has said relatively little about the internal cognitive structure of the inductive process. The modelling tradition — from emergent modelling [4] to the Blum–Leiß cycle [5] — has produced fine-grained cognitive maps of the transitions between situation and formal mathematics, but has only recently been deployed at the interface with specific science disciplines. Chemistry-education research has begun to close this gap [1, 2, 7], but has not yet articulated a concise instructional algorithm that lecturers can apply across topics. ICM contributes such an algorithm by aligning the inductive trigger of [3] with the situation–real–mathematical–results–interpretation–validation sequence of [5] and the model-of / model-for distinction of [4].

Three aspects of the algorithm deserve closer scrutiny. First, the inductive trigger of step 1 places significant demands on the lecturer's selection of phenomena. Phenomena that are too complex overwhelm students' situational reasoning; phenomena that are too trivial fail to motivate the mathematical work of steps 4 and 5. The chemistry-first curriculum design perspective [7] provides useful guidance here, as does the observation in [2] that some chemical contexts (kinetics, thermodynamics) are particularly well suited to motivating specific mathematical concepts (derivatives, integrals).

Second, the mathematisation step (step 4) is the point at which students with weaker mathematical backgrounds are most likely to disengage. The literature on inductive methods emphasises that students must be guided through, not simply left to discover, the abstraction from real to mathematical model [3]. Step 4 of ICM accordingly carries the strongest scaffolding load and benefits from the tiered tasks and small-group discussion formats documented in the broader active-learning literature [3, 6].

Third, the empirical findings in [1] suggest that the back-and-forth between chemistry and mathematics in steps 6 and 7 is not the linear sequence depicted in standard modelling cycles but a deliberation-rich process in which students repeatedly evaluate intermediate results against chemical plausibility. ICM is compatible with this finding: in practice, steps 6 and 7 can be iterated, and earlier steps revisited, whenever an interpretation fails the chemical-plausibility test.

The principal limitation of the present work is its conceptual nature. The seven-step algorithm is grounded in well-established research traditions but has not yet been tested in a coordinated empirical design. Subsequent work should pursue at least three lines of investigation: (i) quasi-experimental comparison of ICM-based and conventional teaching across matched chemistry cohorts, with outcomes measured on conceptual mathematical understanding, modelling competence and chemistry achievement; (ii)

think-aloud or process-tracing studies that examine students' actual trajectories through the seven steps; and (iii) design-based research on the lecturer-level practices and institutional conditions that support sustained ICM implementation.

5. Conclusion

Teaching mathematics to chemistry students is most productive when the abstract content is approached inductively, anchored in authentic chemical contexts, and structured as a modelling cycle that explicitly traverses the transitions between chemical phenomena and mathematical structures. The Inductive Contextual Modelling (ICM) algorithm proposed in this paper integrates these three commitments into a seven-step instructional sequence — chemical phenomenon, situation model, real model, mathematisation, mathematical work, chemical interpretation, validation and generalisation — that lecturers can apply across a wide range of chemical topics. Each step is anchored in established research from inductive teaching [3], emergent modelling [4] and the mathematical modelling cycle [5], and is consistent with the empirical findings of context-based learning [6] and recent chemistry-specific modelling research [1, 2, 7]. The empirical validation of ICM, particularly through quasi-experimental and process-tracing designs, constitutes a promising agenda for further research and a practical resource for the reform of mathematics teaching in chemistry programmes.

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