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The Envelope of a Family of Curves in the Plane

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Abstract

The concept of the envelope of a family of curves plays a significant role in differential geometry, mathematical analysis, and applied mathematics. An envelope represents a curve that is tangent to each member of a given family of curves at some point. This paper presents a systematic study of envelopes of one-parameter families of curves in the plane. Theoretical foundations, analytical methods for determining envelopes, and geometric interpretations are discussed in detail.

Keywords: envelope of curves, family of curves, plane geometry, differential equations, parametric curves

Introduction

The study of families of curves and their envelopes is a classical topic in mathematics, with origins tracing back to the works of Leibniz, Euler, and Lagrange. In plan geometry, a family of curves is typically defined by an equation involving a parameter, and the envelope of this family represents a curve that is tangent to each member of the family at least at one point..

The purpose of this paper is to present a comprehensive and structured analysis of envelopes of families of curves in the plane. The objectives of the study are:

- to define and formalize the concept of an envelope;
- to describe analytical methods for constructing envelopes;
- to provide illustrative examples of typical curve families;

to discuss applications and limitations of the envelope concept.

Methods

Definition of a Family of Curves

A one-parameter family of curves in the plane is commonly represented by an equation of the form

$F(x, y, a) = 0,$

where x and y are Cartesian coordinates, and a is a real parameter. Each fixed value of a defines a distinct curve in the plane.

Analytical Method for Determining Envelopes

$$\tau=2\ (\sigma_t\ (\sigma_t+\sigma'))$$

σ' is the effective normal stress
σt is the tensile strength of the material

$\tau=2\sigma_t+\sigma'\tan(\phi')$ Where, φ' is the angle of friction on the crack surfaces.

This system expresses the condition that the envelope is tangent to a member of the family at the point of contact. The partial derivative condition ensures that nearby curves intersect the envelope at the same point.

Parametric Representation

In many cases, it is convenient to represent the envelope parametrically. Solving the system for x and y in terms of the parameter yields:

$$x = x(a), \quad y = y(a),$$

which defines the envelope as a parametric curve.

Illustrative Examples

Several standard families of curves were analyzed, including:

- families of straight lines;
- families of circles with varying radii;
- families of parabolas and other conic sections.

Symbolic computation and graphical visualization were used to verify analytical results.

Results

Envelope of a Family of Straight Lines

Eliminating the parameter yields the envelope:

y^2 = 4x,

which is a parabola.

Envelope of a Family of Circles

σ′=σ′+σ *m* 3 +*s*

1 3 *ci*| σ |⎝ *ci* ⎠

are the major and minor effective principal stresses at failure is the uniaxial compressive strength of the intact rock material are material constants, where *s* = 1 for intact rock.

σ′+σ′ σ′−σ′| 1 *d*σ′ −1 |

σ′=1 3_1 3 3

n 2 2| *d*σ′ || 1 *d*σ′ +1 |⎝3⎠

Consider the family of circles defined by

(x - a)^2 + y^2 = a^2.

Applying the envelope condition results in a straight line, demonstrating that envelopes may be curves of a different type than the original family members.

Existence Conditions

The results show that not every family of curves admits an envelope. The existence depends on the smoothness of the function F and the solvability of the envelope system. Singular points and degenerate cases may prevent the formation of a well-defined enveloped.

Conclusion

This paper has presented a detailed study of the envelope of a family of curves in the plane. By employing analytical and parametric methods, conditions for the existence and construction of envelopes were established. The results highlight the geometric significance and practical importance of envelopes across various scientific disciplines.

Future research may focus on numerical approaches, envelopes of multi-parameter families, and extensions to surfaces in three-dimensional space.

References

1. Arnold, V. I. (1992). Catastrophe Theory. Springer.
2. Do Carmo, M. P. (2016). Differential Geometry of Curves and Surfaces. Dover Publications.
3. Kreyszig, E. (2011). Advanced Engineering Mathematics (10th ed.). Wiley.
4. Salmon, G. (2005). A Treatise on the Higher Plane Curves. Chelsea Publishing.
5. Spivak, M. (1999). A Comprehensive Introduction to Differential Geometry, Vol. 2. Publish or Perish.
6. Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
7. Thompson, J. M. T. (1982). Instabilities and Catastrophes in Science and Engineering. Wiley.