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Smooth Mappings

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Abstract

This paper considers smooth mappings between Euclidean spaces and their basic differential properties. A mapping defined on an open set is studied through its coordinate functions, differentiability, and continuity. Special attention is given to the Jacobian matrix, the differential of a mapping, and the concept of rank. Definitions of smooth, infinitely differentiable, and continuous mappings are presented, and illustrative examples are discussed to demonstrate the theoretical results.

Keywords

Smooth mapping, Jacobian matrix, differential, rank of a mapping, Euclidean space

Introduction

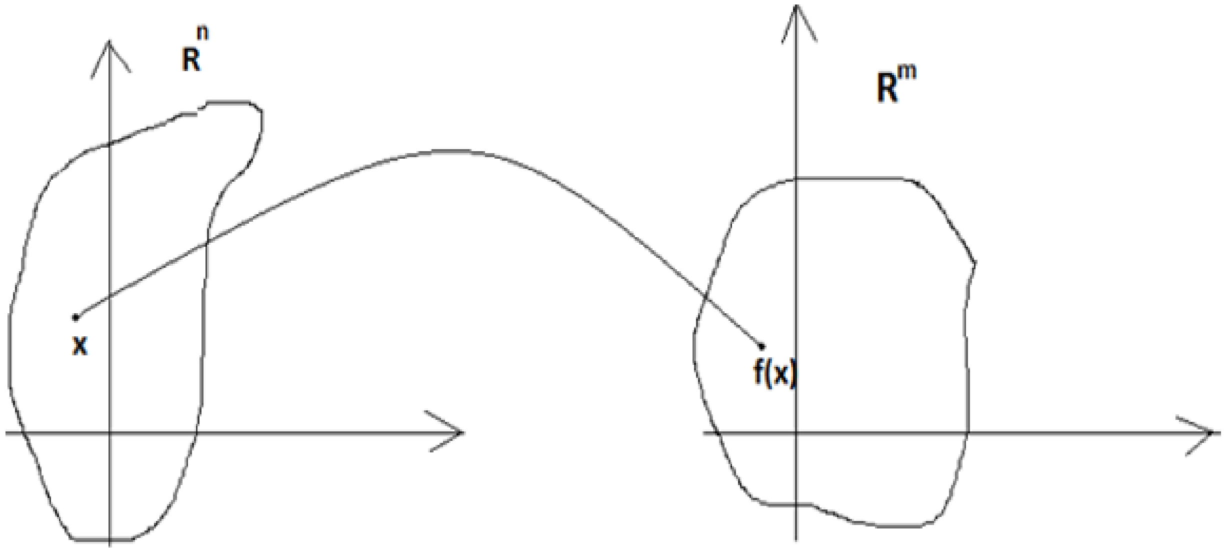
Mappings between Euclidean spaces play an important role in differential geometry and mathematical analysis. Many geometric and analytical properties of such mappings can be studied using partial derivatives and linear approximations. In particular, the Jacobian matrix provides a convenient tool for describing the local behavior of a mapping near a given point. The aim of this paper is to introduce basic definitions related to smooth mappings, differentials, and the rank of a mapping. The paper also illustrates these concepts with simple examples, which help to clarify their geometric meaning and practical applications.

Tetod

Bizga $f: G \to R^m$ akslantirish berilgan bo'lsin. Bu yerda $G \subset R^n$ soha (ochiq to'plam). Bu akslantirish m ta n o'zgaruvchili funksiyalar berilishiga teng kuchlidir:

$$y_1 = f_1(x_1, x_2, ..., x_n)$$
$$y_2 = f_2(x_1, x_2, ..., x_n)$$
$$\dots\dots\dots\dots\dots\dots$$
$$y_m = f_m(x_1, x_2, ..., x_n)$$

1-Ta'rif. **[1]** Berilgan $f : G \to R^m$, $G \subset R^n$ akslantirish uchun $f_1, f_2, ..., f_m$ funksiyalar uzluksiz va r tartibgacha uzluksiz xususiy hosilalarga ega bo'lsa, f akslantirish silliq akslantirish yoki C^r -akslantirish deyiladi. Bu yerda $r \geq 0$



rasmdagi $x = (x_1, x_2, ..., x_n)$ nuqta va uning obrazi $f(x) = (y_1, y_2, ..., y_m)$ nuqta bo'ladi.

Agar barcha f_i funksiyaning $\forall$ tartibli xususiy hosilalari uzluksiz bo'lsa f akslantirish cheksiz ko'p marta differensiallanuvchi deyiladi $f \in C^\infty$. Uzluksiz akslantirish C^0 - akslantirish deyiladi.

2-Ta'rif . [2] Berilgan $f : G \to R^m$ akslantirish uchun

$$J(f)=\begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_m}{\partial x_1} & \frac{\partial f_m}{\partial x_2} & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix} \qquad (2.1.1)$$

matritsa f - akslantirishning Yakobi matritsasi deyiladi. Yakobi matritsasi chiziqli akslantirishni aniqlaydi . Berilgan $x_0 \in R^n$ nuqta va $\upsilon \in T_{x_0} R^n$ vektor uchun

$$w_1 = \left.\frac{\partial f_1}{\partial x_1}\right|_{x_0} v_1 + \left.\frac{\partial f_1}{\partial x_2}\right|_{x_0} v_2 + \ldots + \left.\frac{\partial f_1}{\partial x_n}\right|_{x_0} v_n$$

....

$$w_m = \left.\frac{\partial f_m}{\partial x_1}\right|_{x_0} v_1 + \left.\frac{\partial f_m}{\partial x_2}\right|_{x_0} v_2 + \ldots. + \left.\frac{\partial f_m}{\partial x_n}\right|_{x_0} v_n$$

qoida bo’yicha $w = \{w_1, w_2, \ldots, w_m\}$ vektorni hosil qilamiz. Bu yerda $v = \{v_1, v_2, \ldots, v_n\}$

Bu f akslantirishning x_0 nuqtadagi differensiali deyiladi va $d_{x_0} f$ ko’rinishda belgilanadi.

Bizga $G \subset R^n$ - ochiq to’plam va $f : G \to R^m$ akslantirish berilgan bo’lsin ($C^r, r \geq 1$ sinfga tegishli). f akslantirishning x_0 nuqtadagi rangi deb, (1) formula bilan aniqlangan Yakobi matritsasining rangiga aytiladi va $rankd_{x_0} f$ kabi belgilanadi.

3-Ta’rif . Har bir nuqtada f akslantirishning rangi maksimal ya’ni $\forall x \in G, rankd_{x_0} f = \min(n,m)$ $\forall x \in G, rankd_{x_0} f = \min(n,m)$ bo’lsa bu akslantirishning rangi maksimal deyiladi.

Misol: Quyidagi

$$\begin{cases} f_1(x) = x \\ f_2(x) = x^2 \\ f_3(x) = x^3 \end{cases}$$

formulalar yordamida berilgan $f : R^1 / \{0\} \to R^3$ akslantirishning Yakobi matrisasini va uning $x_0 = 2$ nuqtada rangini topaylik:

$$J(f) = \begin{pmatrix} \frac{\partial f_1}{\partial x} \\ \frac{\partial f_2}{\partial x} \\ \frac{\partial f_3}{\partial x} \end{pmatrix} = \begin{pmatrix} 1 \\ 2x \\ 3x^2 \end{pmatrix}$$

$$rankd_{x_0} f = rank \begin{pmatrix} 1 \\ 4 \\ 12 \end{pmatrix} = 1.$$

Uning hamma nuqtalarda rangi birga teng.

Adabiyotlar

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